CSE4203: Computer Graphics

Chapter – 6 (part - C)

Transformation Matrices

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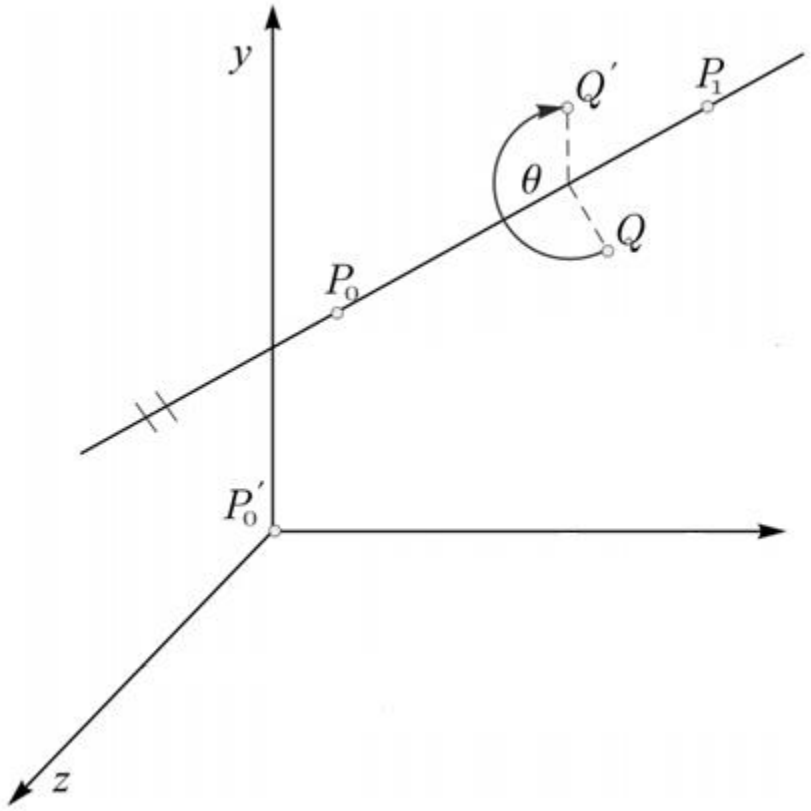
Outline

- 3D Transformation
- Rotation about an arbitrary line

References

- http://ami.ektf.hu/uploads/papers/finalpdf/AMI_40_from175to186.pdf
- http://web.iitd.ac.in/~hegde/cad/lecture/L6_3dtrans.pdf

Rotation about an arbitrary line (1/1)

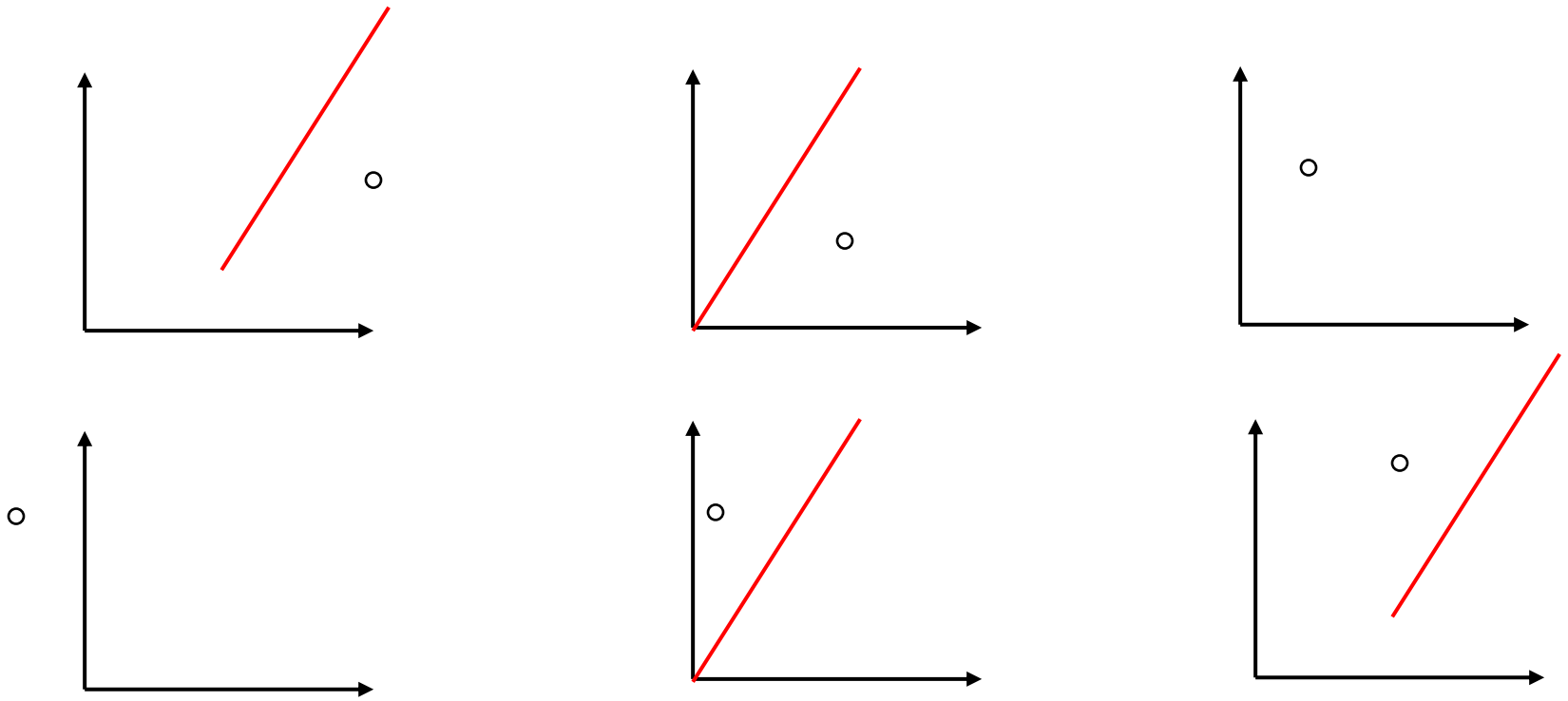


The basic idea is to make the arbitrary rotation axis coincide with one of the principle axis. Assume an arbitrary axis in space passing through the point **P0 (x0, y0, z0)** and **P1 (x1, y1, z1)** .

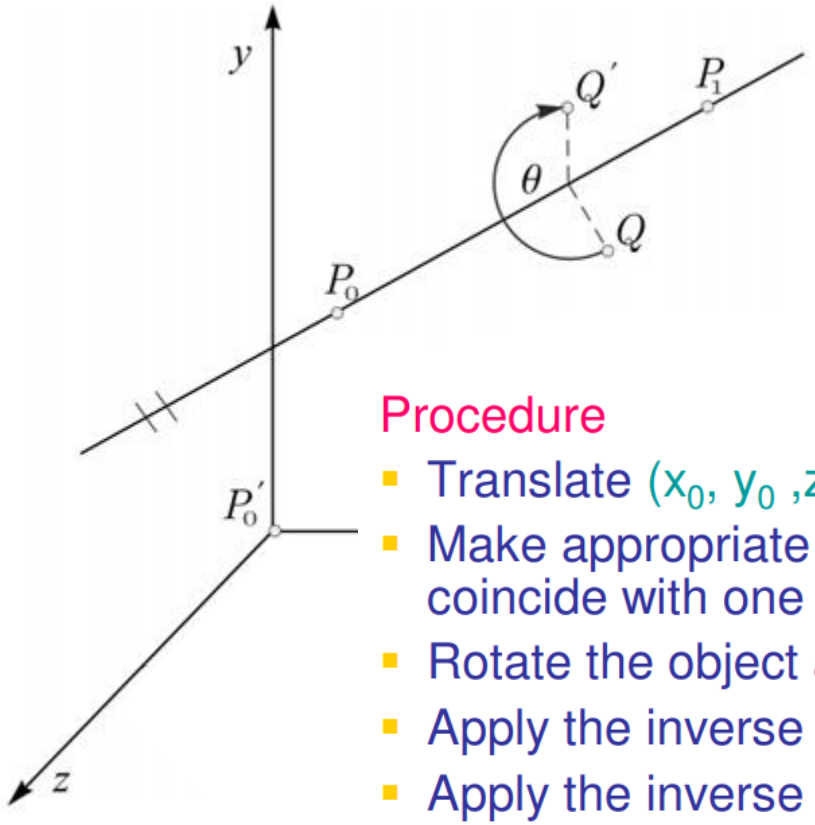
Credit: http://ami.ektf.hu/uploads/papers/finalpdf/AMI_40_from175to186.pdf

In 2D case (1/1)

Reflecting about an arbitrary line



Steps (1/1)

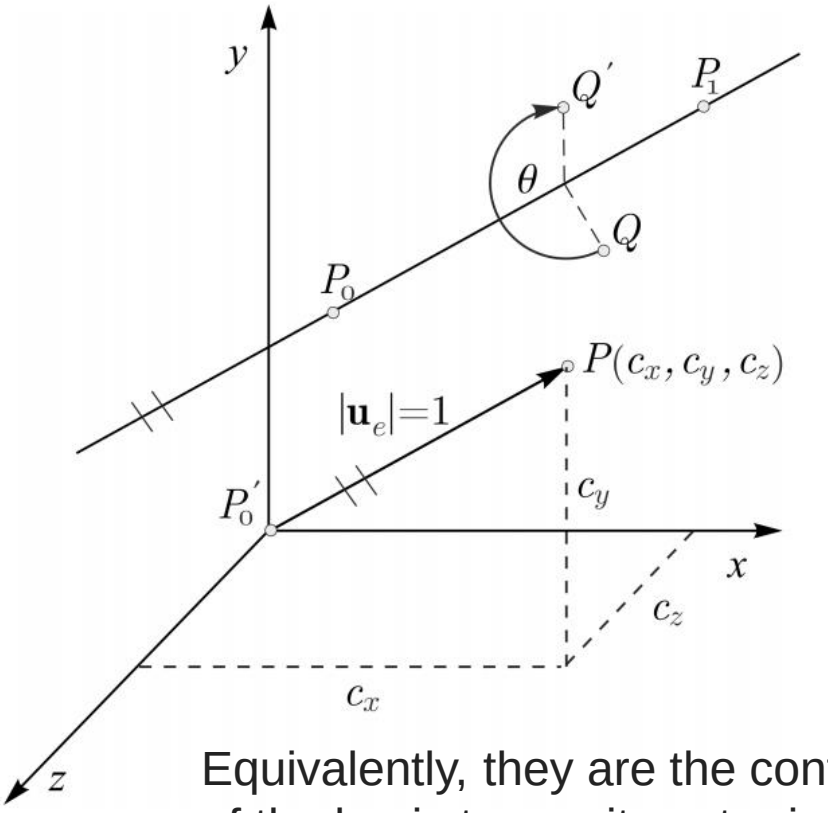


Procedure

- Translate (x_0, y_0 ,z_0) so that the point is at origin
- Make appropriate rotations to make the line coincide with one of the axes, say z-axis
- Rotate the object about z-axis by required angle
- Apply the inverse of step 2
- Apply the inverse of step 1

Credit: http://ami.ektf.hu/uploads/papers/finalpdf/AMI_40_from175to186.pdf | http://web.iitd.ac.in/~hegde/cad/lecture/L6_3dtrans.pdf

Direction Cosine (1/1)



- Direction cosines of a vector are the cosines of the angles between the vector and the three coordinate axes.

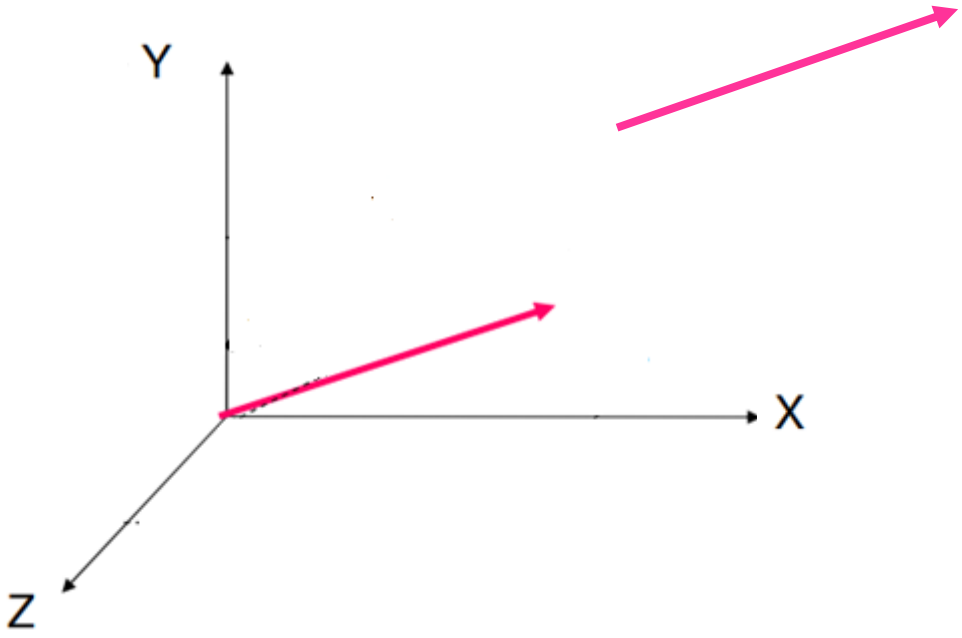
$$\mathbf{u} = P_1 - P_0$$

$$\mathbf{u_e} := \frac{\mathbf{u}}{|\mathbf{u}|} = (c_x, c_y, c_z)$$

Equivalently, they are the contributions of each component of the basis to a unit vector in that direction.

Credit: http://ami.ektf.hu/uploads/papers/finalpdf/AMI_40_from175to186.pdf

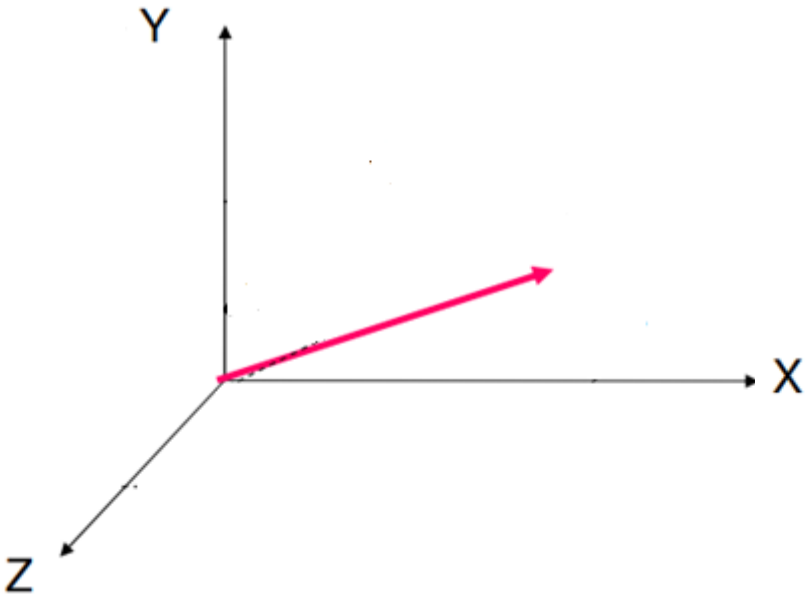
Coinciding the line with Principal axis (1/5)

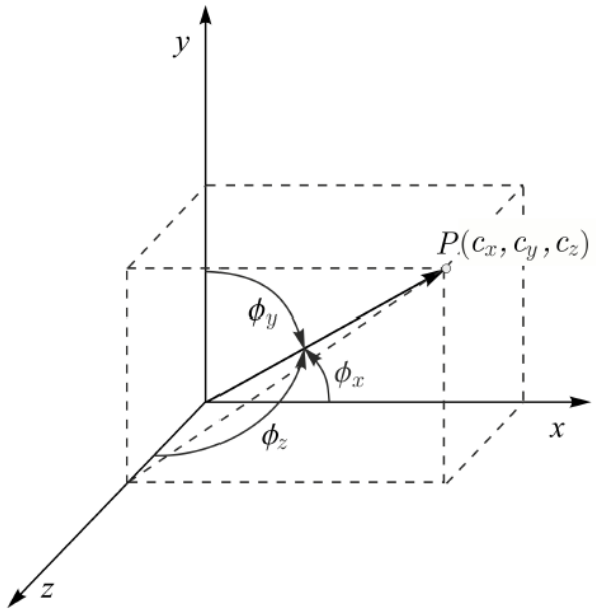


Source: http://web.iitd.ac.in/~hegde/cad/lecture/L6_3dtrans.pdf

Coinciding the line with Principal axis (2/5)

Coinciding the arbitrary axis with any axis
the rotations are needed about other two axes

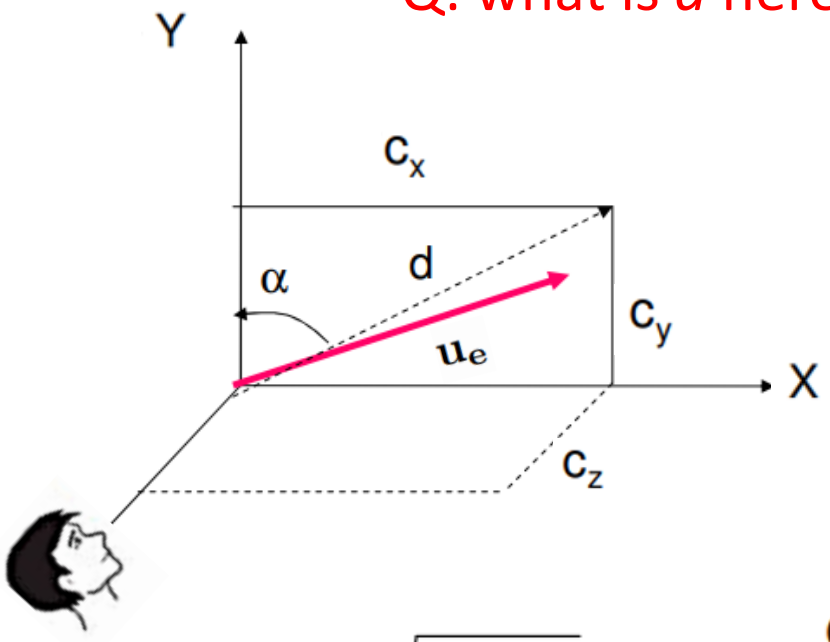


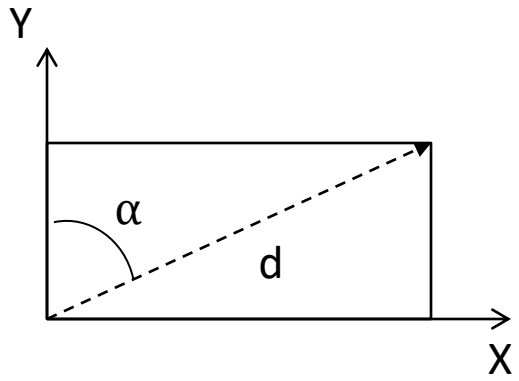


Source: http://ami.ektf.hu/uploads/papers/finalpdf/AMI_40_from175to186.pdf | http://web.iitd.ac.in/~hegde/cad/lecture/L6_3dtrans.pdf

Coinciding the line with Principal axis (3/5)

Q: what is *d* here?

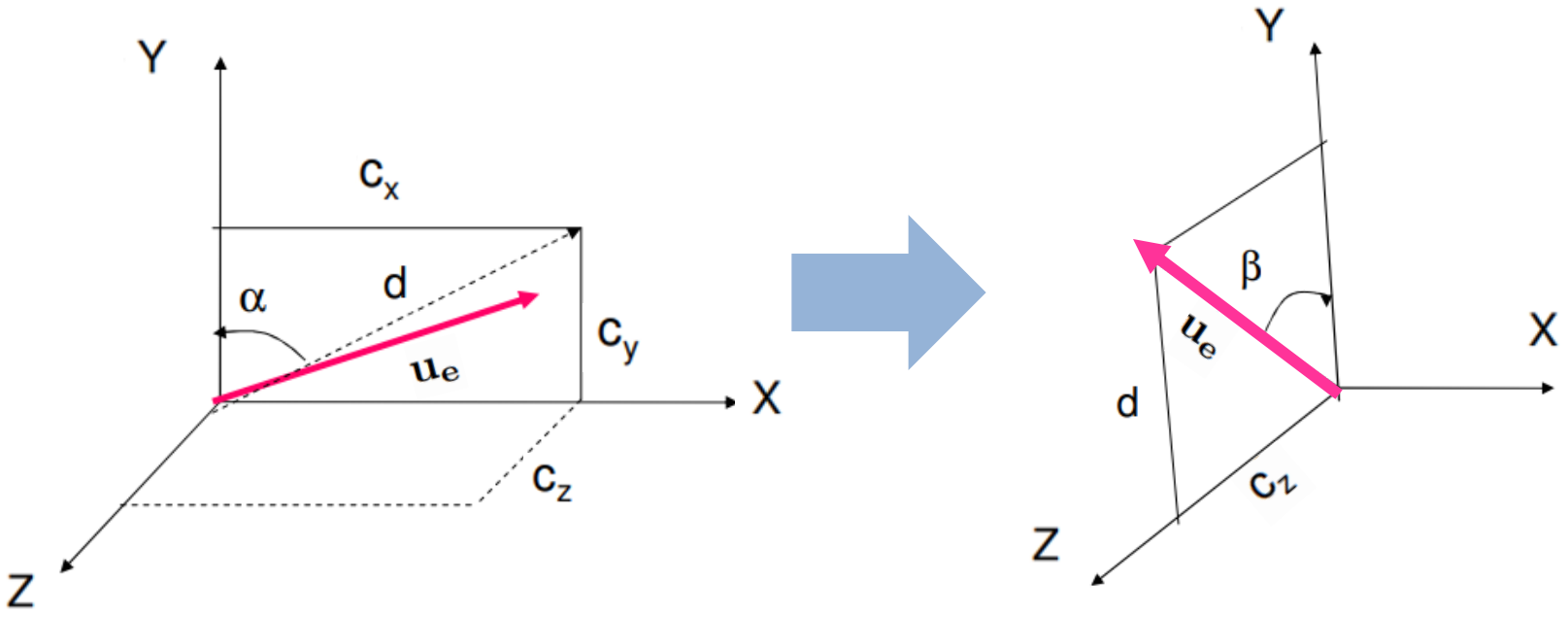




$$d = \sqrt{c_x^2 + c_y^2} \qquad \cos\alpha = \frac{c_y}{d} \qquad \sin\alpha = \frac{?}{d}$$

Source: http://web.iitd.ac.in/~hegde/cad/lecture/L6_3dtrans.pdf

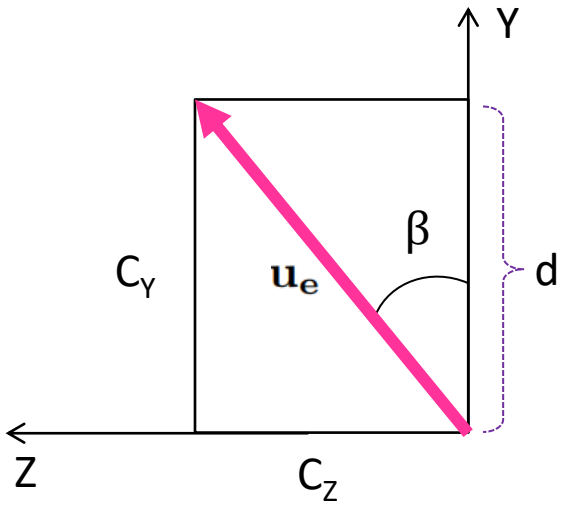
Coinciding the line with Principal axis (4/5)

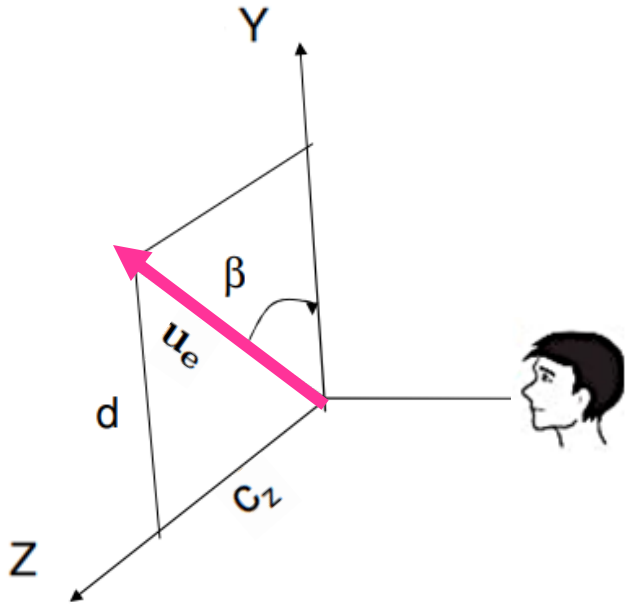


What is the rotation matrix?

Source: http://web.iitd.ac.in/~hegde/cad/lecture/L6_3dtrans.pdf

Coinciding the line with Principal axis (5/5)

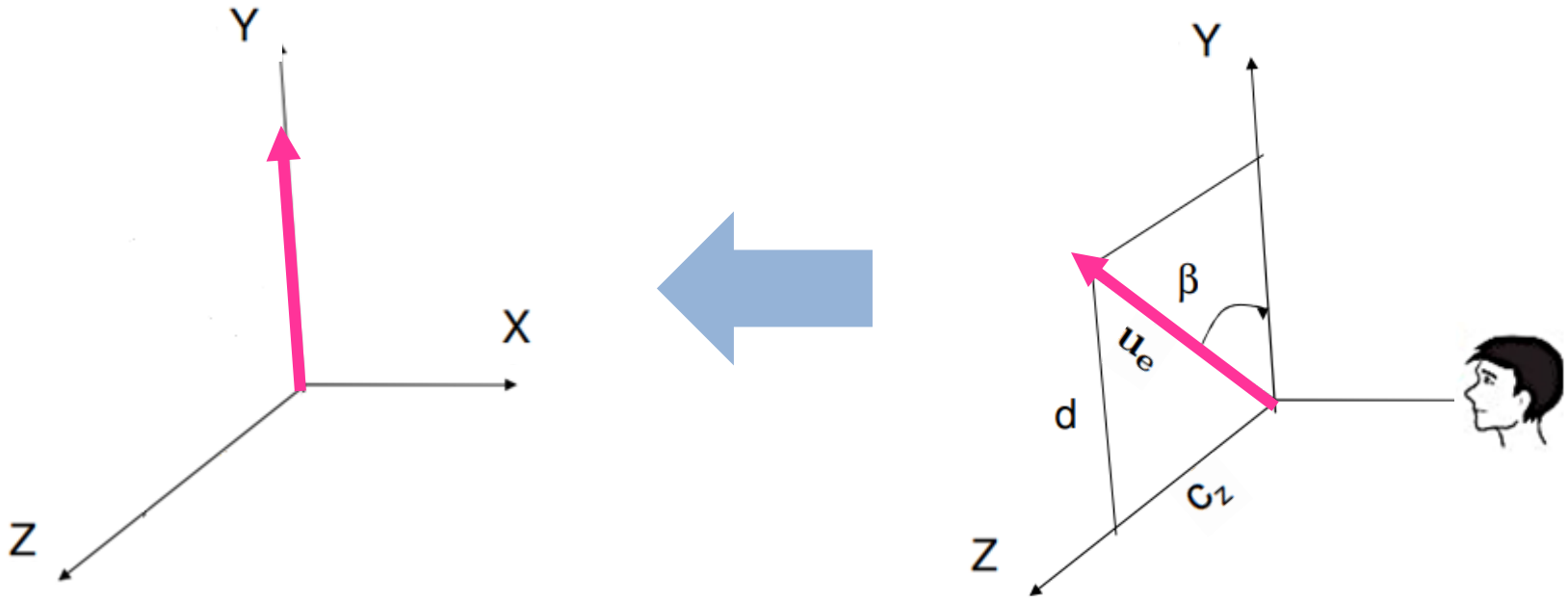




$$\cos\beta = d \qquad \sin\beta = \ ?$$

Source: http://web.iitd.ac.in/~hegde/cad/lecture/L6_3dtrans.pdf

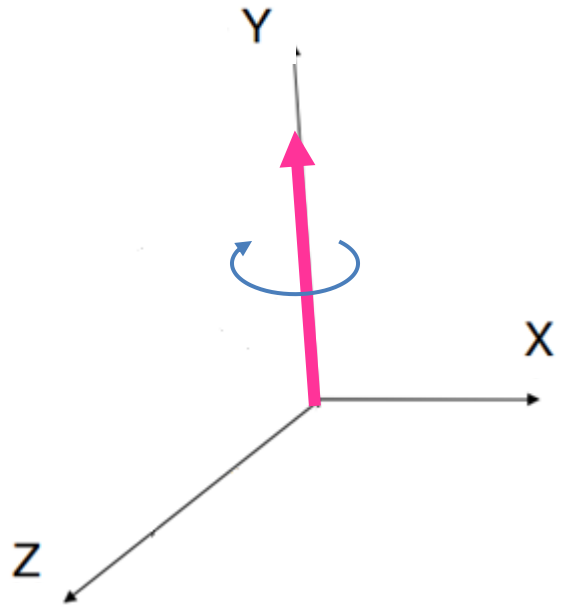
Rotating about the principal axis (1/2)



What is the rotation matrix?

Source: http://web.iitd.ac.in/~hegde/cad/lecture/L6_3dtrans.pdf

Rotating about the principal axis (2/2)



What is the rotation matrix?

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Undoing the steps (1/1)

- Q: What are the undoing steps?

Composite Transformation (1/1)

- **$M = T^{-1} * R_x^{-1}(-\beta) * \ldots . ?$**

- **$Q' = M*Q$**

Practice Problem

- Reflect a point about an arbitrary plane